2026 Fall USTC Graph Theory Homework

Week10

Exercises

Exercise 1. (a) Prove that if G is a simple graph with $\delta \geq \left\lfloor \frac{|V(G)|}{2} \right\rfloor$, then $\lambda(G) = \delta(G)$.

(b) Give an example of a simple graph G with $\delta(G) = \left\lfloor \frac{|V(G)|}{2} \right\rfloor - 1$ and $\lambda(G) < \delta(G)$.

Exercise 2. Let G be a 3-connected graph with $|V(G)| \geq 5$. Prove that there exists $e \in E(G)$ such that $\kappa(G \cdot e) \geq 3$.
(W. T. Tutte, 1961)

Exercise 3. Let $\kappa(D) \geq k$, and let $x_0, x_1, \cdots, x_k$ be any $k+1$ vertices of D. Prove that D contains k internally vertex-disjoint (x_0, x_i)-paths and k internally vertex-disjoint (x_i, x_0)-paths, $i = 1, 2, \cdots, k$.