2026 Fall USTC Graph Theory Homework

Week3[1]

Exercises

Exercise 1. Prove: let D be a simple graph, and $k = \max\{\delta^+, \delta^-\}$, then D contains a directed path of length at least k.

Exercise 2. Let G be a simple graph of order n, and $\omega = \omega(G)$. Prove:

(a) $e(G) \leq \dfrac{1}{2}(n-\omega)(n-\omega+1)$;

(b) If $e(G) > \dfrac{1}{2}(n-1)(n-2)$, then G is connected.

Exercise 3. Let G be a non-bipartite simple graph of order n, $k \geq 2$, and

$$\delta(G) > \left\lfloor \frac{2n}{2k+1} \right\rfloor.$$

Prove that G contains an odd cycle of length $\leq 2k-1$.

[1]The week number follows the academic calendar of the Academic Affairs Office; this course begins from the second week.