2026 Fall USTC Graph Theory Homework

Week9

Exercises

Exercise 1. Prove that there is no convex polyhedron having an odd number of faces each of which has an odd number of edges.

Exercise 2. Prove that every convex polyhedron has at least one triangular face or one vertex that is a trihedral angle.

Exercise 3. Suppose that $N = (D_{xy}, \mathbf{c})$ is a capacity network and $B = (S, \overline{S})$ is an (x, y)-edge cut. Prove that for every (x, y)-flow $\mathbf{f}$ in N,

$$\mathbf{f}(B) - \mathbf{f}^-(S) = \mathbf{f}^+(S) - \mathbf{f}^-(S) = \begin{cases} \operatorname{val}\mathbf{f}, & \text{if } x \in S; \\ -\operatorname{val}\mathbf{f}, & \text{if } y \in S. \end{cases}$$

Exercise 4. Let $N = (D_{xy}, \mathbf{c})$ be a capacity network, let $\mathbf{f}$ be an (x, y)-flow in N, and let $B = (S, \overline{S})$ be an (x, y)-edge cut. Prove that

(a) if $\mathbf{f}(a) = \mathbf{c}(a)$ for every $a \in (S, \overline{S})$ and $\mathbf{f}(a) = 0$ for every $a \in (\overline{S}, S)$, then $\mathbf{f}$ is a maximum flow and B is a minimum cut;

(b) if $\mathbf{f}$ is a maximum flow and B is a minimum cut, then $\mathbf{f}(a) = \mathbf{c}(a)$ for every $a \in (S, \overline{S})$ and $\mathbf{f}(a) = 0$ for every $a \in (\overline{S}, S)$.